

Universality limits on bulk fermions

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Abstract

The Kaluza-Klein fermion excitations induce mixing between the Standard Model fermions and loss of universality. The flavour mixing not present in the Standard Model can be made to vanish aligning the Yukawa couplings and the Dirac masses of the heavy modes, but universality is only recovered when these masses go to infinity. This implies a bound on the lightest new heavy quark, $M_1 \gtrsim 3 - 5$ TeV, which together with the electroweak precision data limits will allow the Large Hadron Collider to provide a crucial test of the Randall-Sundrum ansatz for solving the gauge hierarchy.

Key words: Field Theories in Higher Dimensions, Beyond the Standard Model, Quark Masses and Mixings.

PACS: 11.10.Kk, 12.15.Ff, 12.60.-i

Attempts to solve the gauge hierarchy problem using models with extra dimensions have received a great deal of attention during the past few years [1]. In a proposal by Arkani-Hamed, Dimopoulos and Dvali the hierarchy between the Planck and electroweak scales is related to the large volume of the extra dimensions where only gravity propagates [2]. Randall and Sundrum (RS) proposed an alternative solution based on a non-factorizable geometry with a warped background metric in a slice of AdS_5 [3]. The exponential warp factor, obtained imposing four-dimensional Poincaré invariance, accounts in this case for the hierarchy between the Planck and the electroweak scales (see however Ref. [4]). These models with extra dimensions also give a rich TeV phenomenology. In the RS proposal, only gravity is assumed to propagate in

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the extra dimension. However the possibility of placing the Standard Model (SM) fields in the five-dimensional bulk has been also considered in the literature. Bulk scalars were analysed in Ref. [5], gauge bosons propagating in the bulk were studied in Ref. [6], and fermions were included for the first time in Ref. [7]. The complete SM living in the bulk has been treated in Ref. [8] and a complete parametrization of bulk field masses and their phenomenology can be found in Ref. [9], where supersymmetry is also discussed. Finally, the phenomenology of the RS model with the fields living on and off the wall, is reviewed in Ref. [10], where experimental bounds and the reach of Tevatron and the Large Hadron Collider (LHC) are studied in detail.

Fields living in the five-dimensional bulk can be expanded as a tower of Kaluza-Klein (KK) four-dimensional states, with the mass of each level being the (quantized) momentum in the transverse dimension. In the case of SM fermions, chiral zero modes can be obtained using an orbifold projection in the fifth dimension. The rest of the KK tower, however, is necessarily vector-like: the massive modes are Dirac particles whose Left-Handed (LH) and Right-Handed (RH) parts transform in the same way under the SM gauge group. The presence of these extra fermions in the spectrum can induce large mixing between the SM zero modes. This possibility has been usually neglected arguing the mixing suppression due to the large KK fermion masses M . Indeed experiment tends to banish these KK excitations above $\sim \text{TeV}$, implying at least a suppression of $\frac{v^2}{M^2} \sim \left(\frac{0.25\text{TeV}}{1\text{TeV}}\right)^2 \sim 0.06$, where v is the electroweak vacuum expectation value. However, smaller M masses are also possible. For instance, it can be shown in the RS model with fermions in the bulk that there is a point in parameter space where the conformal limit is recovered [9]. The five-dimensional momentum is then conserved and couplings between zero mode fermions and a non-zero mode gauge boson are forbidden. As a result, bounds on the new gauge boson masses coming from electroweak precision data or direct production of KK gauge boson excitations do not apply. Moreover, around this point fermion couplings to the graviton tower remain small. In summary, the KK excitations could have masses $\sim 0.5 \text{ TeV}$ or even smaller in this region of parameter space, depending on the ratio between the bulk curvature and the Planck mass [10]. Thus, ignoring quark mixing and the corresponding universality constraints, the experimental information available at present and even after LHC leaves an open window to small KK masses. In this paper we discuss the constraints on the mixing induced by bulk fermion excitations. Their contributions to fermion couplings can be readily read from Ref. [11]. There the SM extension with an arbitrary number of vector-like fermions is considered and the effective Lagrangian resulting from integrating them out obtained. Using this Lagrangian we show in the following that to keep the quark mixing in the RS model small enough to fulfil universality to few per cent, the lightest KK quark excitation must be heavier than $3 - 5 \text{ TeV}$ in the window around the conformal point. Vector-like quark masses

will be also constrained by direct production at LHC. As a matter of fact, a lower bound of ~ 1.5 TeV will be placed on them if none of such quarks is observed [12]. However, the universality limit derived from the expected precision in the determination of the top couplings at LHC [13] is larger by more than a factor of 2. This will close the window of small masses, allowing LHC to test crucially the RS ansatz for solving the gauge hierarchy problem. We assume the consistency of this model with SM fields off the boundary. It will be shown that all the mixing contributions of the KK fermions have the same sign, with the total sum being dominated by the first excited states. It seems improbable that other contributions to two-fermion gauge couplings cancel those from KK fermion excitations, which is the largest tree level source for mixing beyond the SM [14].

Let us first review the RS model to fix our notation [3]. The topology of the fifth dimension is an orbifold S_1/Z_2 of radius R , with two 3-branes sitting on the orbifold fixed points ($y = 0$ for the Planck boundary and $y = \pi R$ for the TeV boundary). The background metric which satisfies five-dimensional Einstein's equations and four-dimensional Poincaré invariance reads $ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$, with $\sigma = k|y|$ and $1/k$ the AdS_5 curvature radius. The exponential warp factor in the metric reduces the only fundamental scale, the Planck mass $M_{Pl} \sim \mathcal{O}(k)$, to TeV scales on the $y = \pi R$ boundary, $M_{Pl} e^{-\pi k R} \sim \mathcal{O}(\text{TeV})$ provided $kR \sim \mathcal{O}(10)$. For simplicity we assume that the SM Higgs lives on the TeV boundary, as it is phenomenologically preferred [8–10,15,16]. We consider that the SM fermions live in the bulk since we want to study the effects of their KK excitations. The SM gauge bosons are also allowed to propagate in the fifth dimension to maintain in general gauge invariance. The KK excitations of the gauge bosons can also introduce fermion mixing but this has been discussed elsewhere [17], and we will not study this possibility in detail here.

In five dimensions there are no chiral fermions. Thus five-dimensional fermions Ψ are vector-like and can have a Dirac mass of the form

$$\mathcal{L}_D = -im_\Psi(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L), \quad (1)$$

where Ψ is the sum of the two four-dimensional “chiralities” $\Psi_{L,R} = \pm\gamma_5\Psi_{L,R}$, transforming in the same way under the gauge group. However, in the RS background Ψ_L and Ψ_R must have opposite parities under the Z_2 symmetry $y \rightarrow -y$. This implies that the Dirac mass must present a kink profile and can be parametrized as $m_\Psi = c_\Psi\sigma'$, where c_Ψ is a free parameter determining the location of the zero mode [9]. This symmetry can be used to classify the chirality of the four dimensional states, surviving as zero modes only those with even chirality. In this way a massless chiral spectrum can be generated.

The KK expansion of the fermion fields can be written [7,9]

$$\Psi_{L,R}(x^\mu, y) = \frac{e^{2\sigma}}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Psi_{L,R}^{(n)}(x^\mu) f_n^{L,R}(y), \quad (2)$$

where the expansion coefficients depending on the coordinate transverse to the brane read for the L and R projections ($n \neq 0$)

$$f_n(y) = \frac{e^{\sigma/2}}{N_n} \left[J_\alpha\left(\frac{M_n}{k} e^\sigma\right) + b_\alpha(M_n) Y_\alpha\left(\frac{M_n}{k} e^\sigma\right) \right]. \quad (3)$$

$M_n > 0$ is the mass of the KK excitation $\Psi^{(n)}$; J_α and Y_α are Bessel functions of order $\alpha = |c \pm \frac{1}{2}|$, where the signs \pm correspond to $f^{L,R}$, respectively; and the normalization constants

$$N_n^2 = \frac{1}{\pi R} \int_0^{\pi R} dy e^{2\sigma} \left[J_\alpha\left(\frac{M_n}{k} e^\sigma\right) + b_\alpha(M_n) Y_\alpha\left(\frac{M_n}{k} e^\sigma\right) \right]^2, \quad (4)$$

take in the limit $M_n \ll k$ and $kR \gg 1$ the approximate form

$$N_n \simeq \frac{e^{\pi k R/2}}{\sqrt{\pi^2 R M_n}}. \quad (5)$$

The constant coefficients $b_\alpha(M_n)$ together with the KK masses are calculated using boundary conditions. For even fields

$$b_\alpha(M_n) = -\frac{(\pm c + 1/2) J_\alpha(M_n/k) + \frac{M_n}{k} J'_\alpha(M_n/k)}{(\pm c + 1/2) Y_\alpha(M_n/k) + \frac{M_n}{k} Y'_\alpha(M_n/k)}, \quad (6)$$

and

$$b_\alpha(M_n) = b_\alpha(M_n e^{\pi k R}), \quad (7)$$

with \pm for $f^{L,R}$, respectively. In the limit $M_n \ll k$ and $kR \gg 1$,

$$M_n \simeq (n + \frac{\alpha}{2} - \frac{3}{4}) \pi k e^{-\pi k R}. \quad (8)$$

We omit the corresponding equations for odd fields because we will not use them explicitly here, for odd boundary conditions do not allow massless zero-mode solutions. Besides, their massive modes do not couple to the boundary

and they cannot acquire masses through Yukawa couplings with a boundary Higgs. The coefficients of the zero modes for the even chiralities are

$$f_0^{\text{L,R}}(y) = \frac{e^{\mp c\sigma(y)}}{\sqrt{\frac{e^{(1\mp 2c)\pi kR} - 1}{(1\mp 2c)\pi kR}}}. \quad (9)$$

Although we are considering fields living in the five-dimensional bulk, the AdS_5 space can localize the different KK states. The value of the mass parameter c determines the location of the zero mode. When $c_{L(R)} = \frac{1}{2}$ ($-\frac{1}{2}$) the conformal limit is recovered, the kinetic terms are independent of y and the five-dimensional momentum is conserved. In this case the zero mode is flat. Values of $c_{L(R)}$ greater (smaller) than $\frac{1}{2}$ ($-\frac{1}{2}$) localize the zero mode near the Planck boundary, while values $c_{L(R)} < \frac{1}{2}$ ($> -\frac{1}{2}$) imply that the zero mode is localized near the TeV boundary. The value of the trilinear couplings between the fermion zero modes and the tower of KK gauge bosons also depends on the value of c , being zero for $c_{L(R)} = \frac{1}{2}$ ($-\frac{1}{2}$) and essentially constant for $c_{L(R)} > \frac{1}{2}$ ($< -\frac{1}{2}$) [9]. The non-zero modes are always localized near the TeV brane.

In order to reproduce the SM we consider three quark doublets q_i with even LH parts, three up-type singlets \tilde{u}_i with even RH parts and three down-type quark singlets \tilde{d}_i also with even RH parts. (We use a tilde for the singlets to better distinguish in the following the tower of KK states.) Their opposite chiralities are odd. All quarks live in the bulk and have mass parameters c_i^q , c_i^u , and c_i^d , respectively. The five-dimensional action containing the Yukawa interactions can be written in general

$$S_{\text{Yuk}} = -i \int d^4x \int dy \sqrt{-g} \left[\lambda_{ij}^{u(5)} \bar{q}_i(x, y) \tilde{u}_j(x, y) \tilde{\phi}(x) + \lambda_{ij}^{d(5)} \bar{q}_i(x, y) \tilde{d}_j(x, y) \phi(x) + \text{h.c.} \right] \delta(y - \pi R). \quad (10)$$

Expanding the five-dimensional fields in KK towers and integrating over the fifth dimension, we find after spontaneous symmetry breaking a four-dimensional mass Lagrangian of the form

$$\begin{aligned} i\mathcal{L}_{\text{mass}} = & \sum_{n,m=0}^{\infty} \left[\lambda_{ij}^{u(nm)} \bar{u}_L^{(n)i} \tilde{u}_R^{(m)j} + \lambda_{ij}^{d(nm)} \bar{d}_L^{(n)i} \tilde{d}_R^{(m)j} \right] + \text{h.c.} \\ & + \sum_{n=0}^{\infty} \left[M_i^{q(n)} (\bar{u}_L^{(n)i} u_R^{(n)i} + \bar{u}_R^{(n)i} u_L^{(n)i} + \bar{d}_L^{(n)i} d_R^{(n)i} + \bar{d}_R^{(n)i} d_L^{(n)i}) \right. \\ & \quad + M_i^{u(n)} (\bar{\tilde{u}}_L^{(n)i} \tilde{u}_R^{(n)i} + \bar{\tilde{u}}_R^{(n)i} \tilde{u}_L^{(n)i}) \\ & \quad \left. + M_i^{d(n)} (\bar{\tilde{d}}_L^{(n)i} \tilde{d}_R^{(n)i} + \bar{\tilde{d}}_R^{(n)i} \tilde{d}_L^{(n)i}) \right], \quad (11) \end{aligned}$$

where we have added to Eq. (10) the Dirac masses in Eq. (1). These can always be taken diagonal. The four-dimensional Yukawa couplings are

$$\lambda_{ij}^{u(nm)} = \lambda_{ij}^{u(5)} \frac{v}{\sqrt{2}} e^{\pi k R} \frac{f_{qL}^{(n)i}(\pi R) f_{uR}^{(m)j}(\pi R)}{2\pi R} \equiv \lambda_{ij}^u a_q^{(n)i} a_u^{(m)j}, \quad (12)$$

$$\lambda_{ij}^{d(nm)} = \lambda_{ij}^{d(5)} \frac{v}{\sqrt{2}} e^{\pi k R} \frac{f_{qL}^{(n)i}(\pi R) f_{dR}^{(m)j}(\pi R)}{2\pi R} \equiv \lambda_{ij}^d a_q^{(n)i} a_d^{(m)j}, \quad (13)$$

with

$$v = e^{-\pi k R} v^{(5)} \sim 250 \text{ GeV}, \quad (14)$$

$$\lambda_{ij}^{u,d} = \lambda_{ij}^{u,d(5)} k \frac{v}{\sqrt{2}} \sim \text{SM masses}, \quad (15)$$

$$a_q^{(n)i} = e^{\pi k R/2} \frac{f_{qL}^{(n)i}(\pi R)}{\sqrt{2\pi k R}}, \quad (16)$$

$$a_{u,d}^{(m)j} = e^{\pi k R/2} \frac{f_{u,dL}^{(m)j}(\pi R)}{\sqrt{2\pi k R}}. \quad (17)$$

Notice that the factor $e^{\pi k R}$ in the definition of $\lambda_{ij}^{u,d(nm)}$ is due to the rescaling of the boundary Higgs canonically normalized. Note also that odd fields are zero at the TeV boundary and then the odd chiralities ($q_R, \tilde{u}_L, \tilde{d}_L$) have zero Yukawa couplings for a boundary Higgs. In matrix notation Eq. (11) reads

$$\mathcal{M}^u = \begin{pmatrix} \tilde{u}_R^{(0)} & \tilde{u}_R^{(1)} & \dots & u_R^{(1)} & \dots \\ \bar{u}_L^{(0)} & \lambda_{ij}^u a_q^{(0)i} a_u^{(0)j} & \lambda_{ij}^u a_q^{(0)i} a_u^{(1)j} & \dots & 0 & \dots \\ \tilde{u}_L^{(1)} & 0 & M_i^{u(1)} \delta_{ij} & \dots & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{u}_L^{(1)} & \lambda_{ij}^u a_q^{(1)i} a_u^{(0)j} & \lambda_{ij}^u a_q^{(1)i} a_u^{(1)j} & \dots & M_i^{u(1)} \delta_{ij} & \dots \\ \vdots & \vdots & \vdots & & \vdots & \ddots \end{pmatrix}, \quad (18)$$

and similarly for \mathcal{M}^d .

In order to obtain the effective Lagrangian describing the interactions between the SM quarks, we integrate out the heavy quark excitations. This has been done for generic vector-like quark additions in Ref. [11]. To use the results there, we must first rotate the zero modes,

$$\tilde{u}_R^{(0)i} = (U_R^u)_{ij} \tilde{u}_R^{\prime(0)j}, \quad \tilde{d}_R^{(0)i} = (U_R^d)_{ij} \tilde{d}_R^{\prime(0)j}, \quad q_L^{(0)i} = (U_L^q)_{ij} q_L^{\prime(0)j}, \quad (19)$$

such that

$$(U_L^{q\dagger})_{ik} \lambda_{kl}^u a_q^{(0)k} a_u^{(0)l} (U_R^u)_{lj} = V_{ij}^\dagger m_j^u, \quad (U_L^{q\dagger})_{ik} \lambda_{kl}^d a_q^{(0)k} a_d^{(0)l} (U_R^d)_{lj} = m_i^d \delta_{ij}. \quad (20)$$

In the SM $m_i^{u,d}$ are the quark masses and V the Cabibbo-Kobayashi-Maskawa (CKM) matrix. To order M^{-2} the quark couplings to Z and W^\pm , $X_{ij}^{u,dL,R}$ and $W_{ij}^{L,R}$, respectively, are in the mass eigenstate basis [11] (sums on family indices are understood throughout the paper)

$$X_{ij}^{uL} = \delta_{ij} - \sum_{n=1}^{\infty} \frac{m_{i,nk}^{\prime(1)\dagger} m_{nk,j}^{\prime(1)}}{M_k^{u(n)2}}, \quad (21)$$

$$X_{ij}^{uR} = \sum_{n=1}^{\infty} \frac{m_{i,nk}^{\prime(3u)\dagger} m_{nk,j}^{\prime(3u)}}{M_k^{q(n)2}}, \quad (22)$$

$$X_{ij}^{dL} = \delta_{ij} - \sum_{n=1}^{\infty} \frac{m_{i,nk}^{\prime(2)\dagger} m_{nk,j}^{\prime(2)}}{M_k^{d(n)2}}, \quad (23)$$

$$X_{ij}^{dR} = \sum_{n=1}^{\infty} \frac{m_{i,nk}^{\prime(3d)\dagger} m_{nk,j}^{\prime(3d)}}{M_k^{q(n)2}}, \quad (24)$$

$$W_{ij}^L = \tilde{V}_{ij} - \frac{1}{2} \sum_{n=1}^{\infty} \frac{m_{i,nk}^{\prime(1)\dagger} m_{nk,l}^{\prime(1)}}{M_k^{u(n)2}} \tilde{V}_{lj} - \frac{1}{2} \tilde{V}_{il} \sum_{n=1}^{\infty} \frac{m_{l,nk}^{\prime(2)\dagger} m_{nk,j}^{\prime(2)}}{M_k^{d(n)2}}, \quad (25)$$

$$W_{ij}^R = \sum_{n=1}^{\infty} \frac{m_{i,nk}^{\prime(3u)\dagger} m_{nk,j}^{\prime(3d)}}{M_k^{q(n)2}}, \quad (26)$$

where \tilde{V} is the corrected unitary CKM matrix [11]. The superscripts (1),(2), and (3u),(3d) stand for mixing with heavy vector-like up and down singlets, and up and down quarks within doublets, respectively. We have introduced the definitions

$$m_{nk,j}^{\prime(1)} \equiv \lambda_{kl}^{u\dagger} a_q^{(0)l} a_u^{(n)k} (U_L^q)_{lr} V_{rj}^\dagger = \frac{a_u^{(n)k}}{a_u^{(0)k}} (U_R^u)_{kj} m_j^u, \quad (27)$$

$$m_{nk,j}^{\prime(2)} \equiv \lambda_{kl}^{d\dagger} a_q^{(0)l} a_d^{(n)k} (U_L^q)_{lj} = \frac{a_d^{(n)k}}{a_d^{(0)k}} (U_R^d)_{kj} m_j^d, \quad (28)$$

$$m_{nk,j}^{\prime(3u)} \equiv \lambda_{kl}^u a_q^{(n)k} a_u^{(0)l} (U_R^u)_{lj} = \frac{a_q^{(n)k}}{a_q^{(0)k}} (U_L^q)_{kl} V_{lj}^\dagger m_j^u, \quad (29)$$

$$m_{nk,j}^{\prime(3d)} \equiv \lambda_{kl}^d a_q^{(n)k} a_d^{(0)l} (U_R^d)_{lj} = \frac{a_q^{(n)k}}{a_q^{(0)k}} (U_L^q)_{kl} m_j^d, \quad (30)$$

where the second equalities follow from Eq. (20). These allow to rewrite the X_{ij} and W_{ij} corrections in a more transparent way to analyse flavour mixing

$$X_{ij}^{uL} = \delta_{ij} - m_i^u (U_R^{u\dagger})_{ik} \left[\sum_{n=1}^{\infty} \left(\frac{a_u^{(n)k}}{a_u^{(0)k}} \right)^2 \frac{1}{M_k^{u(n)/2}} \right] (U_R^u)_{kj} m_j^u, \quad (31)$$

$$X_{ij}^{uR} = m_i^u V_{il} (U_L^{q\dagger})_{lk} \left[\sum_{n=1}^{\infty} \left(\frac{a_q^{(n)k}}{a_q^{(0)k}} \right)^2 \frac{1}{M_k^{q(n)/2}} \right] (U_L^q)_{kr} V_{rj}^\dagger m_j^u, \quad (32)$$

$$X_{ij}^{dL} = \delta_{ij} - m_i^d (U_R^{d\dagger})_{ik} \left[\sum_{n=1}^{\infty} \left(\frac{a_d^{(n)k}}{a_d^{(0)k}} \right)^2 \frac{1}{M_k^{d(n)/2}} \right] (U_R^d)_{kj} m_j^d, \quad (33)$$

$$X_{ij}^{dR} = m_i^d (U_L^{q\dagger})_{ik} \left[\sum_{n=1}^{\infty} \left(\frac{a_q^{(n)k}}{a_q^{(0)k}} \right)^2 \frac{1}{M_k^{q(n)/2}} \right] (U_L^q)_{kj} m_j^d, \quad (34)$$

$$W_{ij}^L = \tilde{V}_{ij} - \frac{1}{2} m_i^u (U_R^{u\dagger})_{ik} \left[\sum_{n=1}^{\infty} \left(\frac{a_u^{(n)k}}{a_u^{(0)k}} \right)^2 \frac{1}{M_k^{u(n)/2}} \right] (U_R^u)_{kl} m_l^u \tilde{V}_{lj} \\ - \frac{1}{2} \tilde{V}_{il} m_l^d (U_R^{d\dagger})_{lk} \left[\sum_{n=1}^{\infty} \left(\frac{a_d^{(n)k}}{a_d^{(0)k}} \right)^2 \frac{1}{M_k^{d(n)/2}} \right] (U_R^d)_{kj} m_j^d, \quad (35)$$

$$W_{ij}^R = m_i^u V_{il} (U_L^{q\dagger})_{lk} \left[\sum_{n=1}^{\infty} \left(\frac{a_q^{(n)k}}{a_q^{(0)k}} \right)^2 \frac{1}{M_k^{q(n)/2}} \right] (U_L^q)_{kj} m_j^d, \quad (36)$$

where at this order V can be replaced by \tilde{V} in X^{uR} and W^R . The new contributions correcting the SM values, $X_{ij}^{u,dL} = \delta_{ij}$, $X_{ij}^{u,dR} = 0$, $W_{ij}^L = V_{ij}$, $W_{ij}^R = 0$, are products of three 3×3 matrices, where the second one in square brackets is diagonal and the entries of the other two are combinations of SM masses. The matrix in the middle can be further simplified noting that

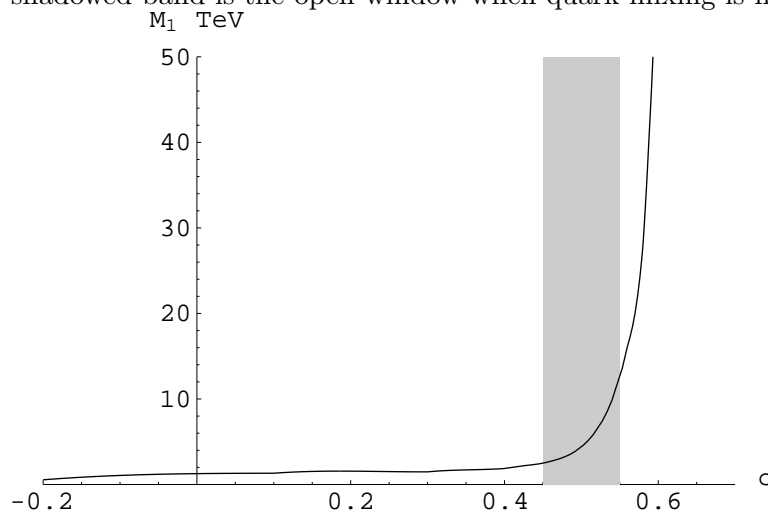
$$a^{(n)} = (-1)^{n-1} a^{(1)}, \quad (37)$$

which, up to a constant, leaves the diagonal elements as an infinite sum of the inverse of the KK heavy masses squared. As can be observed from Eqs. (31-34), there are only two ways to ensure the absence of Flavour Changing Neutral Currents (FCNC), that is to have only diagonal X corrections. One is that the Yukawa couplings are aligned with the Dirac masses, in which case the rotation matrices U are equal to the identity. The other, that the terms in square brackets are proportional to the identity, which can be only accomplished if all the fields of the same type are located at the same point. This means that each type of quark has a common, flavour independent mass parameter, $c_i^{q,u,d} = c^{q,u,d}$.

Let us discuss first the simplest case with all mass parameters equal, $c^q = -c^u = -c^d = c$. Then the square brackets in Eqs. (31-36) only depend on the parameter c for given curvature and warp factor. For $c < \frac{1}{2}$ the ratio of constants $\frac{a^{(1)}}{a^{(0)}}$ is order 1 and the sum is order $\frac{1}{M_1^2}$, the inverse of the squared

mass of the lightest KK heavy mode. For $c \gtrsim \frac{1}{2}$ the ratio grows exponentially (and so does $M_1 \propto ke^{-\pi kR}$ if the value of the square bracket is to remain constant). This is due to the very small zero mode wave function on the TeV boundary [9]. (The same property allows to obtain small neutrino masses in this context [7].) Asking for deviations from the SM value of the diagonal top coupling $X_{tt}^{uL} = 1$ (which is the quantity receiving the largest correction for it is proportional to m_t^2) smaller than few percent, which is the precision to be reached at LHC [13], a limit on the square bracket value in Eq. (31) can be derived. This translates into a limit on $M_1 = M_1^u \simeq 2.45ke^{-\pi kR}$ (where the numerical factor corresponds to $c = \frac{1}{2}$) as a function of c . In Fig. 1 we plot the 90% C.L. M_1 bound assuming that X_{tt}^{uL} is measured at LHC within 5% of its SM value. (Notice the change of notation with respect to Ref. [10]. Our results can be compared replacing c by $-\nu$.) This shows that the narrow window left open after LHC if quark mixing is neglected, $0.45 \lesssim c \lesssim 0.55$ [10], is completely closed with the precise measurement of the diagonal top coupling. In this region the M_1 limit ranges from 2.5 to 12.7 TeV. If the experimental result coincides within 1% with the SM prediction, the M_1 lower bound will vary from 5.5 to 28.4 TeV in the same region. Notice that this bound is independent of the ratio k/M_{Pl} . This is not the case for the bounds coming from direct production of KK graviton states at large colliders. In particular, for $k = 0.01M_{Pl}$ the M_1 lower limit will be $\gtrsim 0.5$ TeV in the region $c \sim 0.5$ if no graviton signal is seen [10]. Direct production of vector-like (KK) quarks will imply $M_1 \gtrsim 1.5$ TeV if such heavy quarks are not observed either [12].

Fig. 1. 90% C.L. limit on the mass M_1 of the first KK quark excitation assuming a deviation of X_{tt}^{uL} from its SM value, 1, smaller than 5% for $c_i^q = -c_i^u = -c_i^d = c$. The shadowed band is the open window when quark mixing is not taken into account.



Other indirect constraints not involving the top quark are less restrictive. Thus, although X_{bb}^{dL} has been measured with a precision of 0.5% [18,19], the

corresponding correction in Eq. (33) is proportional to m_b^2 and the bound on $M_1 = M_1^d$ is reduced by a factor $\sqrt{\frac{0.05}{0.005} \frac{m_b}{m_t}} \sim 0.09$, varying from 0.2 to 1.1 TeV for $0.45 < c = -c^d < 0.55$. Similarly, the unitarity condition $\sum_{j=1}^3 |W_{uj}^L|^2 = 1$ is satisfied to few per mille [19], but the new contributions in Eq. (35) are proportional to m_u^2 and to $\sum_{j=1}^3 |\tilde{V}_{uj}|^2 m_j^{d/2}$, respectively. Hence, the corresponding limits on M_1^u and M_1^d (equal to M_1 for a unique c) are further suppressed by a small mass and by small mixing angles times small masses, respectively.

If each type of quark has a different location, the same limits on $M_1^{u,d}$ above apply. (The bound on M_1^q is equal to the M_1^u limit if X_{tt}^{uR} is measured with the same precision as X_{tt}^{uL} .) Again, as we are only interested in the most stringent bound on the common factor $ke^{-\pi kR}$, the limit on M_1^u from the precise measurement of X_{tt}^{uL} is enough for closing the open window in the RS model with the SM fields off the wall.

If we allow different quarks to be located at different points of the fifth dimension (which could explain the fermion mass hierarchy [9,20]) and then for non-diagonal X corrections, one must wonder about possibly large FCNC and CP violation. These can originate from the exchange of KK gauge bosons [9,17] and from mixing with KK fermions. In the first case the effective scale must be very large (or as above, the first two families must be almost at the same location or in the region of equal coupling, $c_{L(R)} \gtrsim 0.5$ ($\lesssim -0.5$)). We are interested, however, in the effects of quark mixing induced by the tower of vector-like quarks with different family location. In this case FCNC and CP violation are not too large due to the scaling of the X_{ij} corrections with the quark masses $m_i m_j$ (see Eqs. (31-34)). This is enough to suppress those effects below experimental limits if we require X_{tt}^{uL} to agree with its SM value at LHC. Indeed, $|\Delta X_{ij}| \sim \frac{m_i m_j}{m_t^2} |\Delta X_{tt}^{uL}|$, with $\Delta X_{tt}^{uL} = -m_t^2 |(U_R^u)_{kt}|^2 []_k$ and $\sum_{k=1}^3 |(U_R^u)_{kt}|^2 = 1$ and $[]_k$ positive. Then the limits above apply for some k because not all mixing elements vanish. (Similar arguments apply for W_{ij} but with small mixing angles (see Eqs. (35, 36)).) This kind of behaviour has been usually assumed in SM extensions with vector-like quarks (see Ref. [21] and references there in). The models with extra dimensions realize naturally this scaling, introducing an infinite tower of exotic fermions with contributions dominated by the lightest states. A detailed study of FCNC and CP violation and their different origins will be presented elsewhere.

In conclusion, we have shown that if fermions are allowed to live in the bulk of the RS model, mixing effects between the SM fermions and the tower of their KK (vector-like) excitations can be important. In the conformal limit region, this mixing gives a strong constraint, even in the absence of FCNC. At this point, other experimental tests (including both electroweak precision measurements and direct collider searches) provide smaller lower bounds from the decoupling of the towers of gauge bosons and gravitons. Thus, the inclusion of

quark mixing in the phenomenological study of models with extra dimensions results in a significant improvement of the experimental constraints, allowing to completely cover the parameter space up to scales of the order of several TeV.

Acknowledgements

It is a pleasure to thank Ll. Ametller, A. Delgado, M. Masip, M. Pérez-Victoria, A. Pomarol, M. Quirós and F. Zwirner for discussions. JS thanks the Dipartimento di Fisica Galileo Galilei and INFN Sezione di Padova for their warm hospitality. This work has been supported by CICYT, by Junta de Andalucía and by the European Union under contract HPRN-CT-2000-00149. JS thanks MECN for financial support.

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